LIMITING EQUILIBRIUM OF COMPRESSED PLATE WEAKENED BY CIRCULAR HOLE AND CRACKS EXTENDING TO ITS EDGE

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The fracture of brittle porous bodies in uniaxial compression is accompanied by the development of longitudinal cracks caused by stress concentration in the vicinity of the pores. Of interest in this connection is the problem examined herein of the limiting equilibrium of a plate weakened by a circular hole and radial cracks opening circumferentially to the hole contour under the action of compressive stresses at infinity (we do not examine plate stability, assuming it be adequate). The problem is solved within the framework of the δ_k model of a brittle body with cracks [1, 2]. A similar problem for cases of uniaxial and omnidirectional tension was solved by an approximate method in [2]. The development of shear cracks near holes in compressed bodies was examined in [3].

Assume there is a circular hole and two macrocracks extending to its contour in an infinite elastic plate of unit thickness (Fig. 1). The plate is compressed at infinity by a system of external stresses q ($q < 0$), parallel to the alignment of the cracks. Following [2], we represent the elastic stresses in the vicinity of the ends of the crack approximately in the form of the sum

$$\sigma_y(x, 0) \approx \sigma_y^{(0)}(x, 0) + \sigma_y^{(1)}(x, 0) \quad (x \leqslant -a, \; x \geqslant b)$$

Here $\sigma_y^{(0)}(x, 0)$ are the elastic stresses in the plate with circular hole (without cracks) from the action of the compressive stresses $\sigma_x = \sigma$, $\sigma_y^{(1)}(x, 0)$ are the elastic stresses in the plate with rectangular cut along the x axis for $-a \le x \le b$, when the normal pressure $p_n(x) = \sigma_y^{(0)}(x, 0)$ is applied to the edges of this cut on the segments $-a \le x \le -R$ and $R \le x \le b$.

The elastic stresses in a plate with circular hole (without cracks) are expressed as [4]

$$\sigma_y^{(0)}(x, 0) = q\left(\frac{1}{2}\frac{R}{x^2} - \frac{3}{2}\frac{R^4}{x^4}\right) \tag{1}$$

In accordance with [2] the stress $\sigma_y^{(1)}(x, 0)$ is determined from the formula

$$\sigma_y^{(1)}(x, 0) = \frac{1}{\pi\sqrt{(x-b)(x+a)}}\int_{-a}^{b}\frac{p_n(\xi)\sqrt{(b-\xi)(a+\xi)}}{|x-\xi|}\,d\xi \quad \begin{pmatrix} x \leqslant -a \\ x \geqslant b \end{pmatrix} (a \leqslant b)$$

$$p_n(\xi) = \begin{cases} \sigma_y^{(0)}(\xi, 0) & \text{for } -a \leqslant \xi \leqslant -R \\ 0 & \text{for } -R < \xi < R \\ \sigma_y^{(0)}(\xi, 0) & \text{for } R \leqslant \xi \leqslant b \end{cases} \tag{2}$$

The limiting magnitude $q = q^*$ of the external forces can be found from the expression [2]

$$\frac{1}{\pi\sqrt{b+a}}\int_{-a}^{b}\frac{p_n^*(\xi)\sqrt{(b-\xi)(a-\xi)}}{b-\xi}\,d\xi = K_c \tag{3}$$

Here K_c is the Irwin constant [5] and $p_n^*(\xi)$ is found from (2) for the limiting value of the parameters characterizing the external load $q = q^*$.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 10, No. 5, pp. 100-101, September-October, 1969. Original article submitted January 28, 1969.

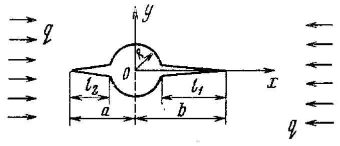

Fig. 1

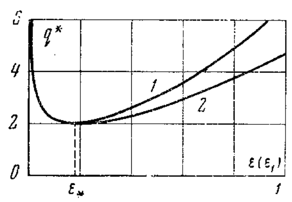

Fig. 2

From (1)-(3) we can obtain

$$q_* = \frac{\pi K_c \sqrt{b+a}}{f(a,b)}$$

$$f(a,b) = \int_{-a}^{-R} \left(\frac{R^2}{2\xi^2} - \frac{3}{2}\frac{R^4}{\xi^4}\right)\left(\frac{a+\xi}{b-\xi}\right)^{1/2} d\xi + \int_{R}^{b} \left(\frac{R^2}{2\xi^2} - \frac{3}{2}\frac{R^4}{\xi^4}\right)\left(\frac{a+\xi}{b-\xi}\right)^{1/2} d\xi \quad (4)$$

After calculating the integrals, we obtain

$$f(a,b) = A(a,b,R)\sqrt{(a+R)(b-R)} - A(a,b,-R)\sqrt{(a-R)(b+R)}$$

$$- B(a,b,R)\ln\frac{[\sqrt{ab} + \sqrt{(a-R)(b+R)}]^2 + R^2}{[\sqrt{ab} + \sqrt{(a+R)(b-R)}]^2 + R^2}$$

$$A(a,b,\pm R) = -\left[\frac{R^2}{b^2}\left(\frac{5}{8} + \frac{1}{8}\frac{b}{a}\right) \pm \frac{R^3}{b^3}\left(\frac{15}{16} + \frac{1}{4}\frac{b}{a} - \frac{3}{16}\frac{b^2}{a^2}\right)\right]$$

$$B(a,b,\pm R) = \frac{R^2\sqrt{ab}}{32a^3b^4}\left[8a^2b^2(a+b) - 3R^2(7a^3 - 3a^2b + 5ab^2 - b^3)\right]$$

If both cracks have the same length ($l_1 = l_2 = l$) and a = b, setting $\varepsilon = l/r$, we obtain

$$q_* = -\frac{\pi K_c}{\sqrt{2R}}\frac{(1+\varepsilon)^4}{\sqrt{2\varepsilon + 3\varepsilon^2 + \varepsilon^3}} \quad (5)$$

If the plate is weakened by a circular hole and a single radial crack (i.e., $l_2 = 0$, $l_1 > 0$) and a = R we obtain

$$q_* = \frac{\pi K_c \sqrt{1+\lambda}}{\sqrt{R(1+\varepsilon_1)}}\frac{1}{f(\lambda)}, \qquad \varepsilon_1 = \frac{l_1}{R}$$

$$\lambda = \frac{R}{b} = \frac{1}{1+\varepsilon_1}, \quad f(\lambda) = A(\lambda)\sqrt{2\lambda(1-\lambda)} - B(\lambda)\ln\frac{1+\lambda}{[1+\sqrt{2(1-\lambda)}]^2 + \lambda} \quad (6)$$

$$A(\lambda) = {}^1/_{16}(\lambda - 14\lambda^2 - 15\lambda^3), \quad B(\lambda) = {}^1/_{32}\lambda\sqrt{\lambda}(11 - 7\lambda + 9\lambda^2 - 21\lambda^3)$$

It can be shown that in the interesting region of $\varepsilon (\varepsilon > 0)$ variation the function $q_* = q_*(\varepsilon)$, defined by (5) and taken in absolute value, has a single minimum (for $\varepsilon = \varepsilon_* = \sqrt{7/5} - 1 \approx 0.18$). Thus, initially (for $\varepsilon < \varepsilon_*$ the crack propagation is unstable and then (for $\varepsilon > \varepsilon_*$) it becomes stable, i.e., increase of the load is necessary for further crack development. Physically this is explained by the fact that in accordance with (1) the stretched zone near the circular hole in the plate has limited extent (corresponding to $\varepsilon \approx 0.732$) and outside this zone there is a zone of small compressive stresses. The variation of the function $q_* = q_*(\varepsilon_1)$, described by (6), has a similar nature. Such transition of the crack from unstable to stable propagation for another problem was noted previously in [6].

Figure 2 shows curves in accordance with (5), (6) of the reduced limit loads $q^* = |q_*| K_c^{-1}\pi^{-1}\sqrt{R}$ as a function of the ratios $\varepsilon = l/R$ and $\varepsilon_1 = l_1/R$. Curve 2 corresponds to two cracks of equal length, curve 1 is for a single crack.

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